Vol 2, No 1, January - June 2017, pp: 71 - 82

Faculty Publications
ISSN: 3008-1017
Title DOI: https://doi.org/10.62458/021024

The Volatility of the CSX Index: GARCH(1,1) Model

Siphat Lim*

ABSTRACT

This study was conducted to predict the conditional variance as well as the volatility of the return of the CSX Index by employing the GARCH(1,1) model with daily data from 19 April 2012 to 12 June 2017. The estimated result of the GARCH(1,1) model which was derived by the maximum likelihood estimation method had revealed that the conditional variance was highly explained by the lagged of square residual as well as the lagged of forecast variance and no ARCH effect was found in this study. The explosive process did not exist, but a mean reverting variance process was detected since the persistence, $\alpha + \beta < 1$. The long-run daily volatility was estimated to be 1.349403% per day or 21.42% per year.

Keywords: CSX Index, GARCH(1,1), Conditional Variance, Volatility.

1. INTRODUCTION

For almost three decades, the need and the use source of funds by investors in Cambodia mainly relied on the banking system. As the Cambodia's economy characterized as a dollarized economy with low amount of saving and relied mainly on the source of fund abroad, the interest rate is very high, two digits, for commercial loan. To mobilize the fund of investment and lower the cost of doing business for domestic investment, the capital market, which is known as the Cambodia Securities Exchange (CSX), has established since 18 April 2012 by the royal government of Cambodia. The first listed company was the Phnom Penh Water Supply Authority (PWSA). The price of the Initial Public Offering (IPO) was KHR6,300 per share and the total IPO shares were 13,045,975. (PWSA, 2017)

Up until now the total listed stocks in the CSX are five: Phnom Penh Water Supply Authority, Grand Twins International (Cambodia) Plc, Phnom Penh Autonomous Port, Phnom Penh SEZ Plc and Sihanoukville Autonomous Port. From the first opening day to present, the average daily of CSX Index is 499.76 and the average daily return is -0.000956, while the average daily volatility is 0.013902.

To predict the volatility of the asset return, one of the most famous models is always use which is called the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. To examine the characteristic and the volatile of the CSX Index, this study is conducted by using the GARCH(1,1) model.

* Siphat Lim, PhD
CamEd Business School
Email: lsiphat@cam-ed.com

2. LITERATURE REVIEWS

As revealed by the estimated result of the GARCH(1,1) model of the Khartoum Stock Exchange (KSE) Index in Sudan, the conditional variance is highly explained by lagged squared disturbance and lagged conditional variance and the sum of the ARCH and GARCH parameters is less than one which suggest a mean reverting variance process in one sub-sample. In addition, the volatility clustering is confirmed by the model in the full and another sub sample since the sum of the ARCH and GARCH parameters is greater than one. According to the ARCH-LM test, the ARCH effect is not found. This shows that the variance equations are well specified. (Ahmed & Suliman, 2011)

To observe the volatility characteristic of the Kenya Stock market, the GARCH model is employed with daily data from January 1, 2008 to October 10, 2010. The estimated of the GARCH(1,1) model has indicated that most of the conditional variance are explained by the previous days forecast variance about 94% and the lagged of squared residual about 4%. The long run average variance per day implied by the model is 86.897% per day. (Koima, Mwita, & Nassiuma, 2015)

The Uganda Securities Exchange (USE) return series exhibits volatility clustering and leptokurtosis as seen by the high excess kurtosis values. Since $\alpha+\beta>1$, the persistence of the GARCH(1,1) model has revealed that the USE returns has an explosive process. (Jalira, Patrick, & W., 2014)

3. METHODOLOGY

3.1. GARCH(1,1) Model

The generalized autoregressive conditional heteroscedasticity, GARCH(1,1) model (Bollerslev, 1986), is specified as follow,
The mean equation is

$$r_t = \mu + \varepsilon_t \tag{1}$$

with $\varepsilon_t = \sigma_t \epsilon_t$ and $\epsilon_t \sim N(0,1)$ independent and identically distributed (i.i.d).
The conditional variance equation is

$$\sigma_t^2 = \gamma V_L + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

where,

r_t	: return of stock index at time *t*,
σ_t^2	: conditional variance at time *t*,
σ_{t-1}^2	: last period forecast, the GARCH term,
ε_{t-1}^2	: lagged square residual, the ARCH term,
V_L	: long-run variance rate,
γ	: weight assigned to V_L,
α	: weight assigned to r_{t-1}^2,
β	: weight assigned to σ_{t-1}^2,

From the model, the weight has to be sum up to one which is,

$$\gamma + \alpha + \beta = 1$$

The GARCH(1,1) model can also be written as,

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

where,

$\omega = \gamma V_L$ and

The long-run conditional variance rate is defined as

$$V_L = \frac{\omega}{1 - \alpha - \beta} \tag{4}$$

The GARCH(1,1) model is the most popular model in conducting a research about the volatility of the return of stock index since the high frequency data could be employed such as daily data as in this research. The lag length of the model is low which might not capture the impact of volatility as well as the return from the previous longer period.

3.2. Data

This study uses daily data of the Cambodia Securities Exchange (CSX) Index from April 19, 2012 to June 12, 2017. The index is extracted from the Cambodia Securities Exchange website: www.csx.com.kh. (CSX, 2017)

The return of the CSX Index is calculated as below:

$$r_t = ln\left(\frac{S_t}{S_{t-1}}\right) \tag{5}$$

where,

r_t	: return of CSX Index at time *t*,
ln	: logarithm,
S_t	: CSX Index at time *t*,
S_{t-1}	: CSX Index at time *t-1*

3.3. Estimating GARCH(1,1) Parameters

To estimate the GARCH(1,1) parameters, the probability distribution of ε_t conditional on the variance is assumed to be normal. The likelihood function (*LF*) has the following form.

$$LF(\omega,\alpha,\beta,\mu|r_1,r_2,\dots,r_T) = \prod_{t=1}^{T}\left[\frac{1}{\sqrt{2\pi\sigma_t^2}}exp\left(-\frac{(r_t-\mu)^2}{2\sigma_t^2}\right)\right]$$

The likelihood function can also be written as,

$$LF(\omega,\alpha,\beta,\mu|r_1,r_2,\dots,r_T) = \frac{1}{\sigma_t^T(2\pi)^T}exp\left(-\frac{1}{2}\sum_{t=1}^{T}\frac{(r_t-\mu)^2}{\sigma_t^2}\right)$$

Take the logarithm of the *LF* to get,

$$lnLF(\omega,\alpha,\beta,\mu|r_1,r_2,\dots,r_T) = -Tln\sqrt{2\pi} - \frac{T}{2}ln\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T}\left(\frac{(r_t-\mu)^2}{\sigma_t^2}\right)$$

$$lnLF(\omega,\alpha,\beta,\mu|r_1,r_2,\dots,r_T) = -\frac{T}{2}ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}ln\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T}\left(\frac{(r_t-\mu)^2}{\sigma_t^2}\right) \quad (6)$$

The calculus is applied to equation (6) in order to find the sample parameters $\hat{\omega},\hat{\alpha},\hat{\beta}$ and $\hat{\mu}$ that maximize the log-likelihood function.

The estimation of the model could be conducted by using software Eviews 8.

3.4. Ljung-Box statistic

The GARCH model appears to have done a good job in explaining the data. For a more scientific test, the Ljung-Box statistic is applied. If a certain series has *n* observations the Ljung-Box statistic is

$$n\sum_{k=1}^{K}w_k\eta_k^2$$

where η_k is the autocorrelation for a lag of *k*, and *K* is the number of lags considered, and

$$w_k = \frac{n+2}{n-k}$$

For *K=15*, zero autocorrelation can be rejected with 95% confidence when the Ljung-Box statistic is greater than 25. (Hull, 2014)

3.5 Future Forecasting

When the GARCH(1,1) model is employed as explained above, the variance rate estimated at the end of day *t-1* for day *t* is

$$\sigma_t^2 = \gamma V_L + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

and from

$$\gamma + \alpha + \beta = 1$$

so that

$$\gamma = 1 - \alpha - \beta$$

from that, GARCH(1,1) can be written as

$$\sigma_t^2 = (1-\alpha-\beta)V_L + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

so that

$$\sigma_t^2 - V_L = \alpha(\varepsilon_{t-1}^2 - V_L) + \beta(\sigma_{t-1}^2 - V_L)$$

On day *t+T* in the future,

$$\sigma_{t+T}^2 - V_L = \alpha(\varepsilon_{t+T-1}^2 - V_L) + \beta(\sigma_{t+T-1}^2 - V_L)$$

The expected value of ε_{t+T-1}^2 is σ_{t+T-1}^2. Thus,

$$E[\sigma_{t+T}^2 - V_L] = (\alpha+\beta)E[\sigma_{t+T-1}^2 - V_L]$$

where *E* is denoted as expectation operator. Using this equation repeatedly yields

$$E[\sigma_{t+T}^2 - V_L] = (\alpha+\beta)^T E[\sigma_t^2 - V_L]$$

or

$$E[\sigma_{t+T}^2] = V_L + (\alpha+\beta)^T(\sigma_t^2 - V_L) \tag{7}$$

4. FINDING

Before estimating the parameters of the GARCH model, the Augmented Dickey-Fuller (ADF) test is applied to the CSX Index as well as the return of the CSX Index in order to check whether each series is stationary or non-stationary. The significance level of 5% is used for the decision rule for this test.

Table 1. Unit Root Test, CSX Index

Null Hypothesis: LOGCSX has a unit root				
Exogenous: Constant				
Lag Length: 0 (Automatic - based on SIC, maxlag=22)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.741064	0.0674
Test critical values:	1% level		-3.435599	
	5% level		-2.863746	
	10% level		-2.567995	
*MacKinnon (1996) one-sided p-values.				

Figure 1. CSX Index, 19 April 2012 to 12 June 2017

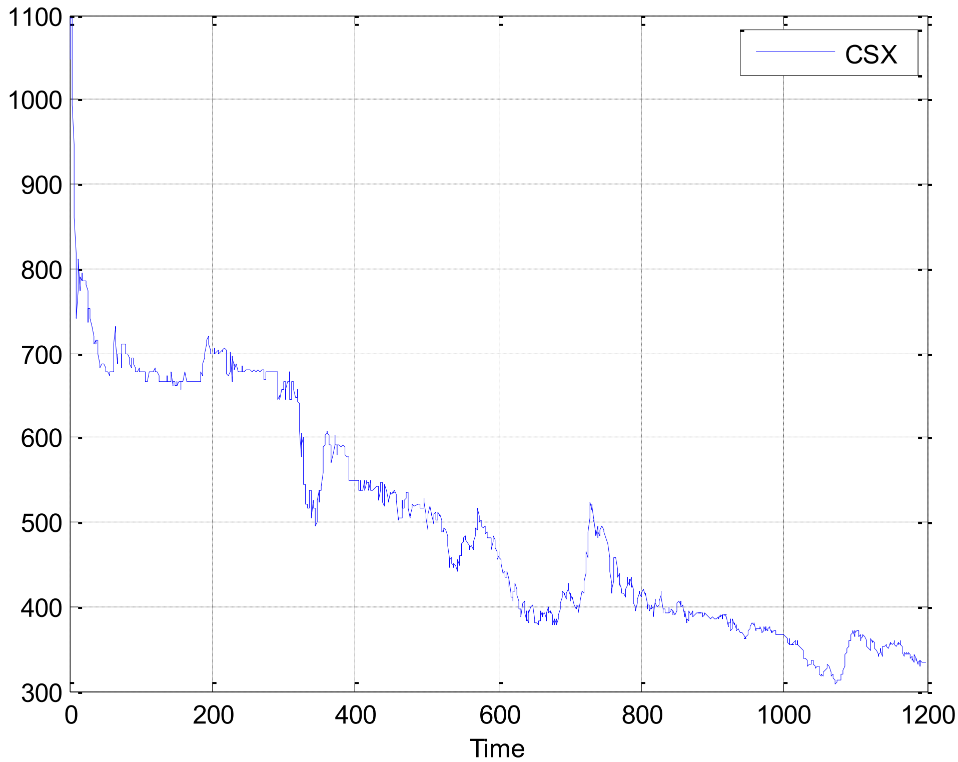
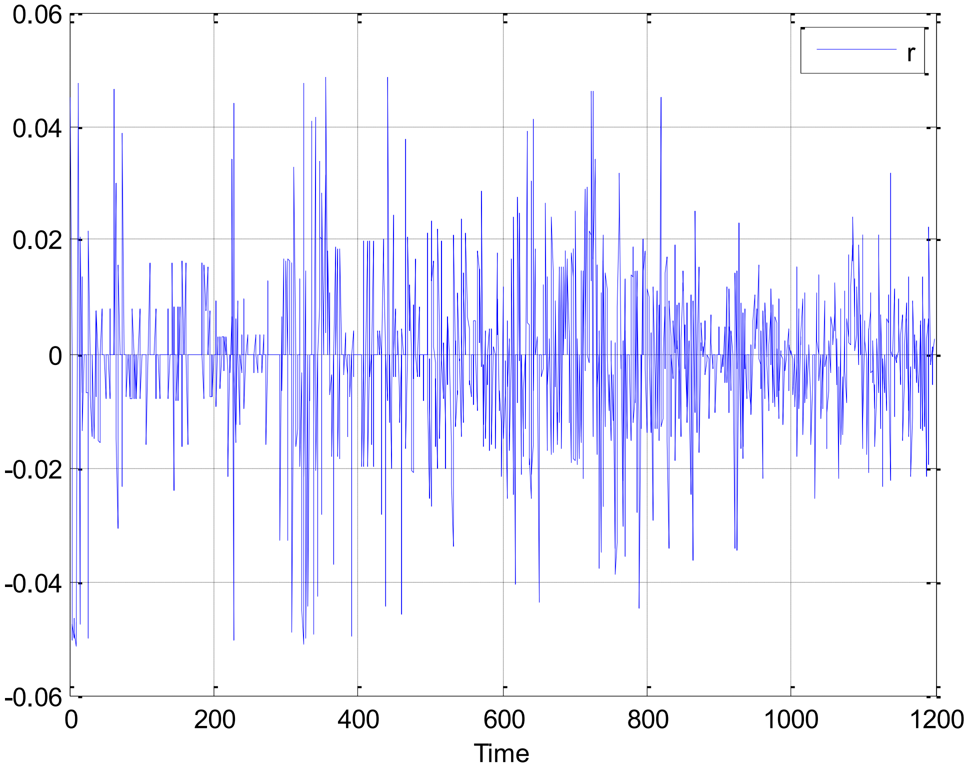


Table 2. Unit Root Test, Return of CSX Index

Null Hypothesis: R has a unit root				
Exogenous: Constant				
Lag Length: 0 (Automatic - based on SIC, maxlag=22)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-34.40433	0.0000
Test critical values:	1% level		-3.435604	
	5% level		-2.863748	
	10% level		-2.567996	
*MacKinnon (1996) one-sided p-values.				

Figure 2. Return of CSX Index, r, 20 April 2012 to 12 June 2017



As indicated in Table 1, the logarithm of the CSX Index has a unit root or non-stationary since the null hypothesis that the series is non-stationary is failed to reject with significance level of 5%, but the return of the CSX Index is stationary because the MacKinnon P-value is very low which is lower than 5% significance level. Thus, the CSX Index is integrated of order one, $I(1)$. The result is more clear when look into the graph of the return series of the CSX Index (see Figure 2), the series exhibit a mean-reverting process which is a kind of stationary process.

Table 3. Descriptive Statistics, CSX Index and Return of CSX Index

	CSX Index	*r*
Mean	499.3056	-0.000956
Median	471.5100	0.000000
Maximum	1096.770	0.048613
Minimum	309.0100	-0.051304
Std. Dev.	136.9871	0.013902
Skewness	0.648502	-0.311380
Kurtosis	2.666646	5.906997
Jarque-Bera	89.44312	440.8182
Probability	0.000000	0.000000
Sum	597668.8	-1.144199
Sum Sq. Dev.	22443483	0.231159
Observations	1197	1197

The estimated result of the GARCH(1,1) model is show as below,
Table 4. Estimated Result, GARCH(1,1) Model

Dependent Variable: R				
Method: ML - ARCH				
Sample (adjusted): 2 1198				
Included observations: 1197 after adjustments				
Convergence achieved after 21 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000733	0.000305	-2.403614	0.0162
	Variance Equation			
C	7.77E-06	9.69E-07	8.016023	0.0000
RESID(-1)^2	0.131678	0.013451	9.789450	0.0000
GARCH(-1)	0.826035	0.013624	60.63089	0.0000
R-squared	-0.000258	Mean dependent var		-0.000956
Adjusted R-squared	-0.000258	S.D. dependent var		0.013902
S.E. of regression	0.013904	Akaike info criterion		-5.972758
Sum squared resid	0.231219	Schwarz criterion		-5.955757
Log likelihood	3578.696	Hannan-Quinn criter.		-5.966353
Durbin-Watson stat	1.972378			

ARCH-LM Test			
F-statistic	0.030423	Prob. F(1,1194)	0.8616
Obs*R-squared	0.030473	Prob. Chi-Square(1)	0.8614

The optimal values of the parameters of the GARCH(1,1) model are revealed as follow: $\widehat{\omega} = 0.0000077$, $\hat{\alpha} = 0.131678$, $\hat{\beta} = 0.826035$ and $\hat{\mu} = -0.000733$ and the maximum value of the log-likelihood is 3578.696. The GARCH(1,1) model is written as
The mean equation:

$$r_t = -0.000733 + \varepsilon_t$$

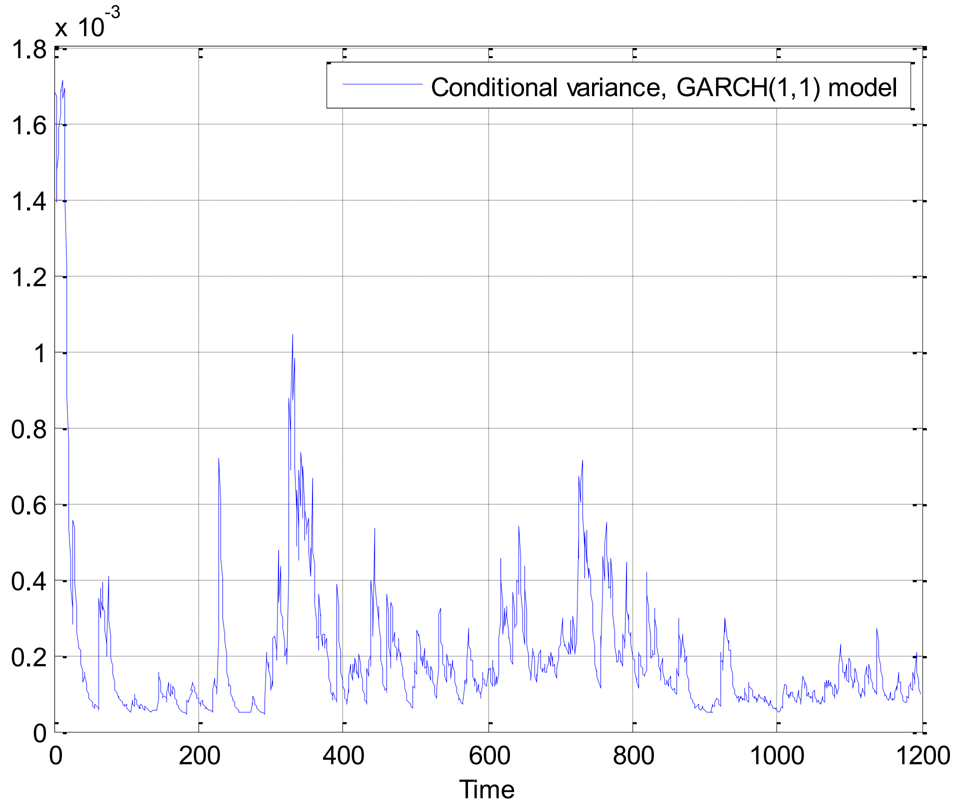
The return of the CSX Index is highly explained by the mean of return since the P-value of the calculated z-Statistic is low.
The conditional variance equation:

$$\sigma_t^2 = 0.0000077 + 0.131678\varepsilon_{t-1}^2 + 0.826035\sigma_{t-1}^2$$

Each slope's P-value of the conditional variance equation are all less than 1% significance level which suggest that the present variance is highly explained by the last period squared residuals, ε_{t-1}^2, from the mean equation and the last period forecast variance, σ_{t-1}^2. In addition, the volatility of the CSX return is not an explosive process since the persistence, $\alpha + \beta < 1$.

The conditional variances can be predicted as indicated in Figure 3.

Figure 3. Conditional Variance, GARCH(1,1) Model



The long-term conditional variance rate is

$$V_L = \frac{\omega}{1-\alpha-\beta} = \frac{0.0000077}{1-0.131678-0.826035} = 0.000182089$$

by that the long-term volatility is $\sqrt{0.000182089} = 0.01349403$ or 1.349403% per day, while the long-term volatility is $0.01349403 * \sqrt{252} = 0.2142$ or 21.42% per year.

Table 5. Ljung-Box before and after GARCH

	Ljung-Box	**Ljung-Box**
Time lag	**before GARCH**	**after GARCH**
1	0.102801887	3.40597E-05
2	0.115840918	0.000231571
3	0.096728413	0.000613439
4	0.066243618	0.000636619
5	0.087959041	0.001503015
6	0.041480572	0.00065433
7	0.034298075	0.00019175
8	0.03033351	0.000956947
9	0.007554682	0.001317728
10	0.018931787	4.97374E-05
11	0.007701054	0.000354656
12	0.020544897	1.10147E-05
13	0.003637157	0.002286072
14	0.001525618	0.001815229
15	0.018789436	0.000436445

For time lag equal 15, zero autocorrelation can be rejected with 95% confidence when the Ljung-Box statistic is greater than 25. The Ljung-Box statistic before GARCH as indicated in Table 5 is about 783.28 which is greater than 25. This is strong evidence of autocorrelation, but after GARCH, the Ljung-Box statistic is reduced to around 13.27, suggesting that the autocorrelation has been largely removed by the GARCH model. In addition to the Ljung-Box statistic, the ARCH-LM test is also conducted in this study and the result of the test as indicated in Table 4 has revealed that the null hypothesis of no ARCH effect is failed to reject with 5% level of significant.

The forecasting of future variance can be made using equation (7),

$$E[\sigma_{t+T}^2] = V_L + (\alpha+\beta)^T(\sigma_t^2 - V_L)$$

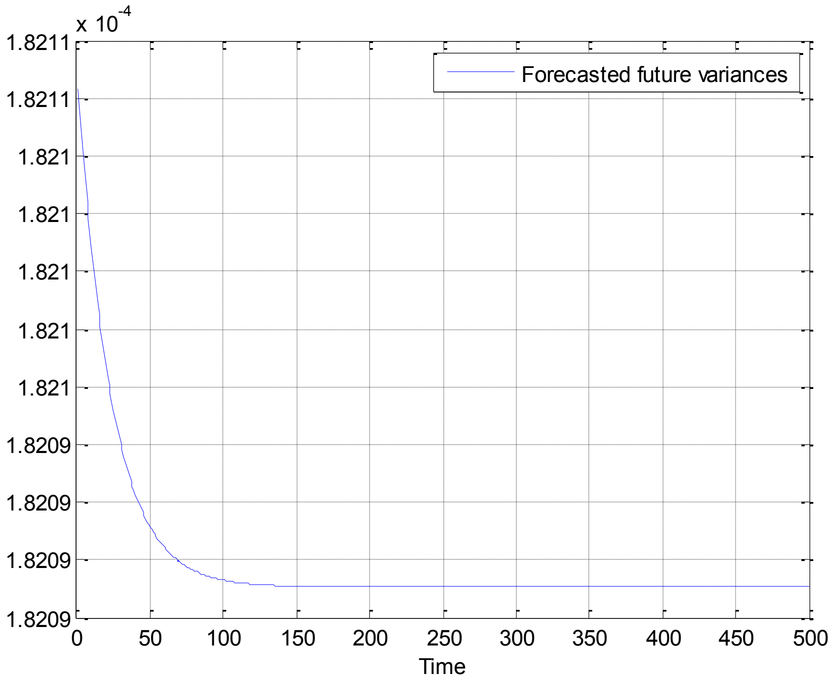
and the result of the forecasting of future variances and volatilities of each time period are shown in Table 6.

Table 6. The forecasting of future variance and volatility per day

Days (*T*)	10	30	50	100	500
Expected variance rate per days	0.000182101	0.000182094	0.000180911	0.0001820893	0.0001820891
Expected volatility per day	1.349447%	1.349422%	1.349411%	1.349404%	1.349403%

The expected volatility per day in the next ten days is 1.349447% which is slightly greater than long-run volatility, 1.349403% per day. The expected variance rate in 500 days is 0.0001820891 and the expected volatility per day is 1.349403% which is converged to the long-run volatility per day.

Figure 4. Expected path for the variance rate



5. CONCLUSION

The main purpose of this study is to investigate the volatility of the return of the CSX Index by using GARCH(1,1) model with daily data from 19 April 2012 to 12 June 2017. In the mean equation of the estimated GARCH(1,1) model has indicated that the return of the CSX Index is highly explained by the mean of the return. The estimated result of the conditional variance equation has shown that the conditional variance is highly explained by the lagged squared residual as well as the lagged forecast variance and the ARCH effect does not exist as revealed by the ARCH-LM test. The explosive process is not found in this study since the sum of the slope of lagged square residual and lagged forecast variance in the conditional variance equation is less than one, $\alpha + \beta < 1$. The long-run daily volatility is predicted to be 1.349403% per day or 21.42% per year.

The result of this study is consistence with the result in the Khartoum Stock Exchange (KSE) Index in Sudan that the conditional variance is highly explained by lagged squared disturbance and lagged forecast variance and the sum of the ARCH and GARCH parameters is less than one which suggest a mean reverting variance process in one sub-sample, but inconsistence with another sub-sample and in the case of Uganda since $\alpha + \beta > 1$. In addition, the long-run daily volatility, 1.349403% per day, is considered to be low as compare to the case of Kenya, 86.897% per day.

BIBLIOGRAPHY

Ahmed, A. M., & Suliman, S. Z. (2011). Modeling Stock Market Volatility using GARCH Models Evidence from Sudan. *International Journal of Business and Social Science*.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics*, 307-27.

CSX. (2017, June). *Cambodia Securities Exchange*. Retrieved from www.csx.com.kh.

Hull, J. C. (2014). *Options, Futures, and Other Derivative.* New Jersey: Pearson Education, Inc.,.

Jalira, N., Patrick, W. G., & W., C. M. (2014). Modelling Volatility of Stock Returns: Is GARCH(1,1) Enought? *International Journal of Sciences: Basic and Applied Research*, 216-223.

Koima, J., Mwita, P., & Nassiuma, D. (2015). Volatility Estimation of Stock Prices using GARCH Method. *European Journal of Business and Management*.

PWSA. (2017, June). *Phnom Penh Water Supply Authority*. Retrieved from http://www.ppwsa.com.kh/en/.